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IMAGE SUPER RESOLUTION BASED ON FINITE RATE INVERSION

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ABSTRACT

Image super-resolution is the process to generate output (high-resolution images) as from input (low-resolution images). In this paper we presents a novel approaches for the super-resolution of image reconstruction from their low-frequency k-space samples, recovering the super resolution version of a given low-resolution image. In that first stage, estimate a resolution-independent mask whose zeros represent the edges of the image. Our work extending the theory of sampling signals of finite rate of innovation (FRI) to two-dimensional curves. We enable its application to IMAGE by proposing extensions of the signal models allowed by FRI theory, more robust to noise and efficient means to determine the edge mask. The second stage of the scheme, we recover the super-resolved image using the discredited edge mask as an image prior. Evaluate present scheme on simulated single-coil image obtained from analytical phantoms, and compare against total variation reconstructions. The proposed algorithm had shown improved performance in both noiseless and noisy settings.

Index Terms—Finite Rate of Innovation, 2Dimensional curves, Super-resolution, K-samples.

I. Introduction

Now a day's Super-resolution (SR) image reconstructions are accelerated, very active area of research. Image super-resolution is the process to generate high-resolution images as from low-resolution images input. The availability of high-resolution Image can greatly facilitate early diagnosis by enabling the detection and characterization of subtle and clinically significant lesions [1]. However, the recovery of very high-resolution Image is often challenging, mainly due to the slow nature of IMAGE acquisition, subject motion, and the rapid decrease in SNR

with resolution. For example, it is common practice to acquire low-resolution data in spectroscopic imaging in consideration of accretion higher k-space samples comes with a heavy signal to noise ratio forfeiture, imaging metabolites at very low concentrations. At the same time, the use of low spatial resolution results in the leakage of strong signals such as water and fat to other spatial regions, thus distorting the metabolite signals.

Several super-resolution and off-the-grid methods were introduced recently to estimate parametric signals with finite number of unknowns from their low-frequency samples. For example, the locations and amplitudes of a finite number of Dirac delta functions probably approximated in destination, its low frequency Fourier samples using continuous total variation minimization [2], or by estimating an annihilation filter and then determining its roots [3]. Unfortunately, the representation using finite number of basic functions is not efficient for the representation of piecewise polynomial images. For example, the gradient of a piecewise constant image is nonzero on a curve, which cannot be represented as a finite linear combination of Dirac delta functions. Recently, Pan et al. introduced a complex analytic signal model for continuous domain two-dimensional signals, whose derivatives are supported on a curve [4].

Specifically, they assume the curve is the zero level-set of a function band limited to a rectangular region in the Fourier domain. The authors showed that the Fourier transform of the curve model will annihilate the Fourier coefficients of the signal derivatives. This property enables them to extend the FRI model [3] to multidimensional signals. The main focus of this paper is to extend the multidimensional FRI model [4] to enable super-resolution IMAGE. While the multidimensional FRI model [4] is very powerful, it has some limitations that restrict its direct applicability to IMAGE. First of all, the complex analytic signal model introduced in [4] is too restrictive and is not applicable to most images. We generalize the signal model to piecewise polynomial and harmonic functions, which are better suited to represent practical signals. The annihilation conditions central to the scheme are not exactly satisfied in the presence of model mismatch and noise. A Cadzow iterative procedure was used in [4] to de-noise the data before estimating the curve coefficients, approach is competition stringent and feels necessity for some knowledge of the underlying model order.

We introduce a novel algorithm based on averaging vectors in the null space of the equations that is robust to noise and model mismatch, and is computationally efficient. Present algorithm follow two steps: in that, first step estimate a super-resolved spatial mask whose zeros

correspond to the edges in the image. Second, we discretized the mask at the desired resolution and use the discrete spatial weights in a weighted total variation. We compare the efficacy of this two step strategy against classical discrete total variation regularized recovery of numerical phantoms from their exact Fourier samples. Fig – 1 shows an example of our method.



Fig – 1 Super resolution of butterfly image reconstruction with scaling factor 3, Right: image of proposed method, Left: low resolution image.

II. SUPER-RESOLUTION OF EDGE IMAGES

In the present work certain circumstances it is potential to super resolve the edges of an image from its low-frequency Fourier samples. In general, unless we assume certain constraints in geometry, we cannot get a hope to recover the edge set. As in [4] we assume the edge sets to be the zero set of a band limited periodic trigonometric polynomial $\mu(\mathbf{r})$ on $[0, 1]^2$,

$$C : \underbrace{\sum_{\mathbf{k} \in \Lambda} \hat{\mu}[\mathbf{k}] e^{j2\pi \langle \mathbf{k}, \mathbf{r} \rangle}}_{\mu(\mathbf{r})} = 0$$

Where Λ is any finite index set, and $\mu[\mathbf{k}]$ are any complex coefficients. We command comparable sets C trigonometric curves. As noted in [4], the set of trigonometric curves have a rich topology, and for large enough bandwidth, they can approximate arbitrary curves to any desired accuracy. As a first approximation, we consider images that are piecewise constant, meaning the image can be expressed as finite linear combinations of functions of the form

$$1_{\Omega}(\mathbf{r}) = \begin{cases} 1 & \text{if } \mathbf{r} = (x, y) \in \Omega \\ 0 & \text{else,} \end{cases}$$

Where Ω is a simple region in $[0, 1]^2$ with piecewise smooth boundary $\partial\Omega$ given as $\{\mu = 0\}$ for some μ as in (1). The Fourier transform of 1_{Ω} is given as

$$\begin{aligned}
\widehat{1_\Omega}(\omega) &= \int_{\Omega} e^{-j\langle \omega, \mathbf{r} \rangle} d\mathbf{r} = -\frac{1}{j\omega_x} \int_{\Omega} \partial_x \left[e^{-j\langle \omega, \mathbf{r} \rangle} \right] d\mathbf{r} \\
&= -\frac{1}{j\omega_x} \oint_{\partial\Omega} e^{-j\langle \omega, \mathbf{r} \rangle} dy, \text{ or } , \\
&= \frac{1}{j\omega_y} \oint_{\partial\Omega} e^{-j\langle \omega, \mathbf{r} \rangle} dx,
\end{aligned}$$

Where the last two equations follow by Green's theorem. Under the Fourier domain relations.

$$\partial_x 1_\Omega(\mathbf{r}) \leftrightarrow -j\omega_x \widehat{1_\Omega}(\omega), \quad \partial_y 1_\Omega(\mathbf{r}) \leftrightarrow -j\omega_y \widehat{1_\Omega}(\omega)$$

The partial derivatives of 1_Ω can be interpreted (in a distributional sense) as a continuous stream of Dirac's in the spatial domain supported on $\partial\Omega$. Thus, formally, we should have $\mu \partial_x 1_\Omega = \mu \partial_y 1_\Omega = 0$, and so we say μ acts as an annihilating mask for the partial derivatives of 1_Ω .

This can be established rigorously in the Fourier domain to give the following:

Proposal 2.1. Let 1_Ω be as above with boundary $\partial\Omega$ given as a trigonometric curve $C: \{\mu(\mathbf{r}) = 0\}$. Then the Fourier transforms of the partial derivatives of 1_Ω are annihilated by convolution with $\mu b[k]$, that is

$$\sum_{\mathbf{k} \in \Lambda} \widehat{\mu}[\mathbf{k}] \widehat{f}(\omega - 2\pi\mathbf{k}) = 0, \quad \text{for all } \omega \in \mathbb{R}^2,$$

$$\text{where } \widehat{f}(\omega) = -j\omega_x \widehat{1_\Omega}(\omega) \text{ or } \widehat{f}(\omega) := -j\omega_y \widehat{1_\Omega}(\omega).$$

The above proposition shows that in principle it is possible to recover the edge set $C: \{\mu = 0\}$ of 1_Ω by solving the linear system of equations (4) for μb .

This is provided we have a sufficient number of low-pass samples to make the system (4) determined. We investigate methods for solving for the filter coefficients μb in the following section. We now consider the case where the image is assumed to be piecewise linear, meaning it can be written as a linear combination of functions of the form

$$g(\mathbf{r}) = \begin{cases} L(\mathbf{r}) & \text{if } \mathbf{r} = (x, y) \in \Omega \\ 0 & \text{else} \end{cases}$$

where L is any affine function $L(\mathbf{r}) = \mathbf{a}^T \mathbf{r} + b$, for $\mathbf{a} \in \mathbb{C}^2$, $b \in \mathbb{C}$. Intuitively, any second derivative of g should vanish except on $\partial\Omega: \{\mu(\mathbf{r}) = 0\}$, where it will act like the derivative of a Dirac. Accordingly, we can show $v = \mu^2$ is an annihilating mask for any second derivative of g , since both v and $\nabla v = 2\mu\nabla\mu$ vanish on $\partial\Omega$. The following proposition expresses this fact in the Fourier domain.

III. FINITE RATE OF INNOVATION

FRI can be defined as a signal with a parametric representation, with a finite ρ given below. Another useful concept is that of a local rate of innovation over a window of size τ , defined as:

$$\rho_\tau(t) = \frac{1}{\tau} C_x \left(t - \frac{\tau}{2}, t + \frac{\tau}{2} \right),$$

Note that $\rho_\tau(t)$ clearly tends to ρ as τ tends to infinity. Given an FRI signal with a rate of innovation ρ , we expect to be able to recover $x(t)$ from ρ samples (or parameters) per unit time. The rate of innovation turns out to have another interesting interpretation in the presence of noise: it is a lower bound on the ratio between the average mean-squared error (MSE) achievable by any unbiased estimator of $x(t)$ and the noise variance, regardless of the sampling method [12]. In image processing, a lot of signals are not band limited and thus cannot be represented using decomposition on band limited filters. Typical illustrations of this are streams of Dirac or piecewise polynomials. Thus, to encode such signals using linear techniques like wavelet decompositions results in many detail coefficients and are therefore not optimal. To circumvent this problem, some authors [1] remarked that streams of Dirac and piecewise polynomials are associated with a finite degree of freedom. For instance, the former are defined by the location of the Dirac's and their amplitude, and are said to be with finite rate of innovation (FRI). The key issue to determine the FRI is to use an annihilator filter (also called locator) filter [1]. Then a filter is applied onto the signal depending on the computed FRI bearing in mind that signal reconstruction has to be still possible from the filtered signal. However, the approach proposed in [1], is limited in that it is very sensitive to noise or other perturbation, and also because most signals are usually not characterized by a finite degree of freedom. For that reason, many developments have been carried out to estimate FRI in a noisy context [2-3] which can be viewed as attempts to stabilize the seminal algorithm.

The various types of moments can be obtained by a linear combination of the geometric moments which therefore constitute the basic elements of moment-based analysis. With an image acquisition system, the observed view $f(x, y)$ is not available so the true moments $m_{p,q}$ of

the continuous function $f(x, y)$ cannot be directly computed. Instead, they are approximated from the acquired image g using the discretized version of

$$m_{p,q} = \sum_{m,n} g[m, n] (mT)^p (nT)^q.$$

When the resolution of g gets low, the discrete moments do not provide a good approximation of the continuous moments and this discrepancy can degrade the performance of any moment-based techniques dramatically. An alternative solution might be to deconvolve each image first and then evaluate the discrete moments on the deconvolved samples. This approach may improve the end result but does not solve the problem when the resolution is low. In [10], new sampling results were proposed for 1-D and 2-D FRI signals. In particular, it is shown that it is possible to compute the exact moments of an FRI signal from its sampled version, provided that the sampling kernel satisfies the String–Fix conditions. In this paper, we propose to use these results on real images in order to extract the true *continuous moments* of a real object f from its samples. The continuous moments are obtained by linear combination of the samples with the coefficients $c_{m,n}^{(p,q)}$ as follows:

$$\begin{aligned} m_{p,q} &= \int \int f(x, y) x^p y^q dx dy \\ &\stackrel{(a)}{=} \int \int f(x, y) \sum_m \sum_n c_{m,n}^{(p,q)} \\ &\quad \times \varphi(x/T - m, y/T - n) dx dy \\ &= \sum_m \sum_n c_{m,n}^{(p,q)} \int \int f(x, y) \\ &\quad \times \varphi(x/T - m, y/T - n) dx dy \\ &\stackrel{(b)}{=} \sum_m \sum_n c_{m,n}^{(p,q)} g[m, n] \end{aligned}$$

Where (a) and (b) follow, respectively, from (2) and (1). Thus, the proposed combination of the samples with $c_{m,n}^{(p,q)}$ allows the extraction of the exact moments from a sampled version of the observed continuous scene. Once the continuous geometric moments are obtained, other types of continuous moments (e.g., central or complex).

In this paper we emphasize on super-resolution using only the LR input image without any external dataset or exemplar image. Our contribution is two-fold. Firstly, we propose a framework to learn the reverse mapping from a LR image to the corresponding HR image pixel wise, relying on local structures defined by each pixel’s neighborhood. Pixels in the HR image are first estimated by their nearest neighbors (repressors) in an initial up sampled image via Gaussian process regression (GPR). The result is then de blurred by learning from LR/HR patches obtained from its down sampled version and the LR input.

3.1 GPR for Super-resolution

In our regression-based framework, patches from the HR image are predicted pixel wise by corresponding patches from the LR image. GPR provides a way for soft-clustering of the pixels based on the local structures they are embedded. Given the intuitive interpretation of hyper parameters of the covariance function. In our case, we can optimize their values through margin a like hood maximization.

3.2 Single Image SR

Figure shows a chain graph representation of GPR for image SR in our setting, where each 3×3 patch from the input image forms a predictor-target training pair. Thus in Equation the observed is the intensity of the pixel at the center of a 3×3 patch and x is an eight-dimensional vector of its surrounding neighbors. In order to adapt to different regions of the image, it is partitioned into fixed-sized and overlapped patches (e.g., 30×30), and the algorithm is run-on each of them separately. The patch-based results are then combined to give the whole HR image. We predict the HR image using a two-stage coarse-to-fine approach, which is consistent with the image formation process. As shown the imaging process can be modeled as a degradation process of the continuous spatial domain depending on the camera’s Point Spread Function (PSF). After discretization, this process can be further expressed as

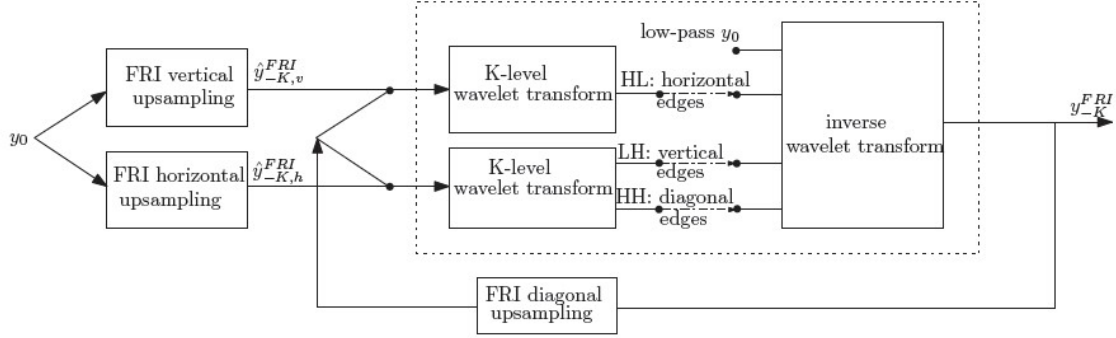
$$\begin{aligned} L &= (f * H) \downarrow^d, \text{ or equivalently,} \\ \tilde{H} &= f * H \text{ and } L = \tilde{H} \downarrow^d \end{aligned}$$

Where L and H denote the LR and HR image respectively, H denotes the blurred HR image, f is the blur kernel and \downarrow^d denotes the down sampling operator with a scaling factored.

3.3 Image up-sampling

Equipped with the resolution enhancement method of the previous section, we now approach image up sampling by modeling lines (along different directions) of images as 1-D piecewise

smooth functions and extend the method of 1-Dcase to 2-D images. For clarity and simplicity, we denote the image at original low-resolution with y_0 and its up sampled version by factor $2K$ with $y \square K$. The low-resolution image y_0 of size $N _ N$ is the low-pass version of a K -level 2D wavelet transform applied to the high-resolution image $y \square K$ of size $2KN _ 2KN$ with all the high-pass coefficients discarded.



3.4 Algorithm for pseudo codes of the SMSR-based image super resolution

INPUT: A low resolution image y and a total scaling factor d .

I. Initialization

- Set the initial parameters $\lambda_1, \lambda_2, \delta$ and c ;
- Through exploiting the multi - scale similarity redundancy , the input LR image y is enlarged to obtain $X_S M$ by the multi - step magnification scheme ;
- Set the initial value of the target HR image that is d times the size of y by down sampling the enlarged result $X_S M$;

II. Outer loop (Dictionary learning & Sparse coding) for each iterations $t=1$ to T_1 do

- Update the dictionaries $\{\phi_k\}$ by means of k - means clustering and PCA ;

- Compute the transform function f with the reference gradient histogram h_T , and update the HR image $\hat{X}^{(t)}$ by the gradient regularization ;
- Inner loop : for each iteration $j=1$ to T_2 do
 1. Update $\hat{X}^{(t+1/2)}$ by the fidelity constraint ;
 2. Compute the sparse coding coefficients of each patch $\hat{\alpha}_i^{(t+1/2)} = \phi_k^T R_i \hat{X}^{(t+1/2)}$,
 Where ϕ_k is the dictionary assigned to the patch $\hat{X}_i = R_i \hat{X}^{(t+1/2)}$;
 3. Compute the regularization parameter and the non local means β_i of $\alpha_i^{(t+1/2)}$;
 4. Update the coding coefficients $\alpha_i^{(t+1)}$ again by the iterative shrinkage operator using ;
 5. Reconstruct the estimate $\hat{X}^{(t+1)}$ using ;
 6. Update the HR image $X_H = \hat{X}^{(t+1)}$.

OUTPUT: A high resolution image X_H .

IV. RESULTS

We applying our proposed methods on both generic and face images to get super resolution results. In our experiment, the input low resolution images by a scaling factor 3, in the proposed method of FRI algorithm formulates the original image as an input is reconstructed by super

resolution sparse technique involves. Result analysis of proposed scheme with various standard input images i.e., reference images are shown as follows.

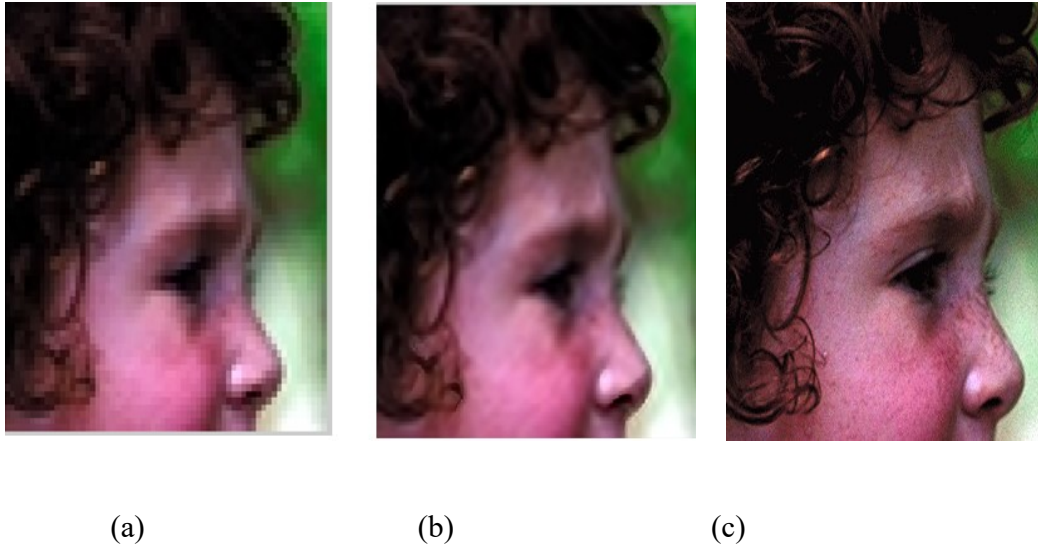


Fig – 2: Results of our fri method shown in table 1 ,girl image magnify by a factor of 3, from right to left our method, bicubic intrpolation, input.

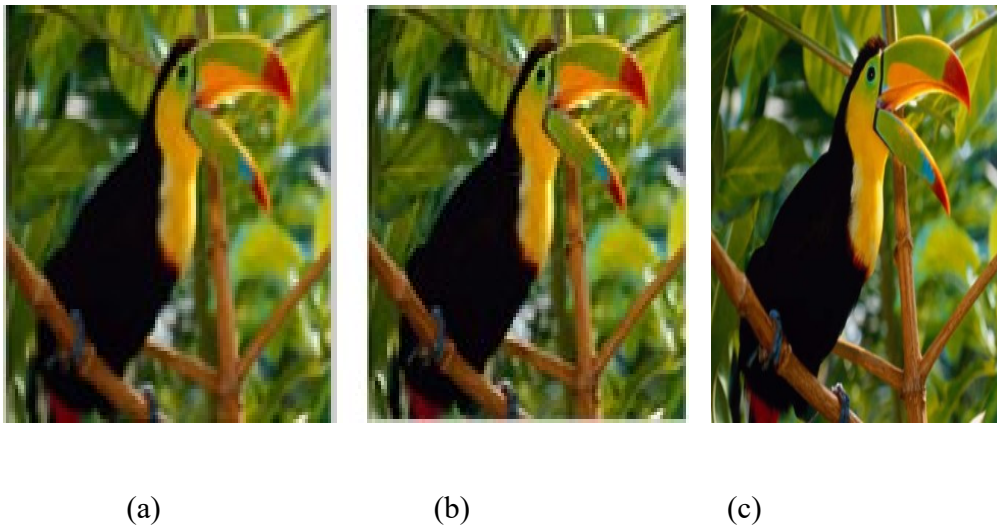


Fig – 3: Results of our fri method shown in table 1,bird image magnify by a factor of 3, from right to left our method, bicubic intrpolation, input.



(a)

(b)

(c)

Fig – 4: Results of our fri method shown in table 1, bird image magnify by a factor of 3, from right to left our method, bicubic intrpolation, input.

Table – 1 . Results of our method			
Images	FRI method		
	PSNR(db)	RSME	Processing time(sec)
Girl face	99.00	0.00	180
Butterfly	23.3265	0.8571	130
Bird	23.7556	0.7547	140

TABLE .2. OBJECTIVE COMPARISON OF THE IMPLEMENTED SR METHODS [19].

Implemented model	PSNR(db)	RMSE	Processing time(sec)
Standard interpolation	23.57	0.7475	0.002
SR nearest neighbor	27.51	0.8832	9.057

SR shift and add	27.09	0.8701	0.081
SR bilinear	28.00	0.8844	0.333
SR Delaunay	28.61	0.903	0.104
SR Delaunay bicubic	29.02	0.9116	0.112
SR iterative back projection	26.90	0.8625	0.373
SR near optimal	26.30	0.835	0.208
SR MAP	28.56	0.9017	0.527
OUR METHOD	99.00	0.00	180

V. CONCLUSIONS

We have presented in this paper two novel approaches for feature extraction that take maximum advantage of the prior knowledge of the acquisition filter and that are based on the basic principles behind the sampling of FRI signals. The first proposed method allows the exact retrieval of the continuous moments of an object from its sampled image. The second one bring back the current location of local image features. These are then used to retrieve the exact location of corner points which are utilized for the exact registration of low-resolution images like in the context of image super-resolution. Experimental results on artificially sampled images and natural images show the efficiency of the proposed feature extraction methods and the validity of the proposed acquisition model.

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