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MATHEMATICAL MODELING OF A ROBOTIC MANIPULATOR

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ABSTRACT

The nonlinear mathematical model of an industrial robot manipulator with four degrees of freedom is developed. For creation of the equations of robot manipulator is applied the matrix method and Lagrange equations of the second kind. The robot manipulator is designed to create automated complexes for service of devices, for removing and installing the equipment, change of details and tools. The kinematic structure of the robot manipulator and the dynamic characteristics of motion for robot manipulator are defined. The mathematical model is presented by nonlinear system of four ordinary differential equations of second order. The analytical solution of the nonlinear system is obtained by method of polynomial transformations. The power of actuators and generalized forces for move the gripping device at a given point of space for a certain period of time are defined.

INTRODUCTION

The robots manipulators are widely used in many fields and spheres of human activity: assembly production, machining, heat treatment. The industrial robot manipulator is an automatic machine, which includes the manipulator with four degrees of freedom. The robot manipulator performs functions similar to the human hand and is controlled by operator or automatically. In the industrial production are used the technological manipulators for operations of assembly and welding. The auxiliary robots manipulators are used for maintenance of the main technological equipment for lifting and transporting [1-4]. The structure of robot manipulator consists of main elements, connected with each other: the base, the stand, the mechanical arm, the gripping device. The working body of the manipulator is the gripping device. The elements connecting the base with the working body constitute a kinematic chain of the robot manipulator. The two adjacent elements constitute a kinematic pair. The number of degrees of freedom provides the movement of the device of gripping in any point of the working area. The main function of the robot manipulator is determined by kinematic scheme.

NONLINEAR MATHEMATICAL MODEL OF THE INDUSTRIAL ROBOT MANIPULATOR

The industrial robot manipulator M20P for robotic complex is considered. The robot manipulator is shown in Figure 1 consists of a base, the stand, hands, the gripping device and actuators for moving and turns.

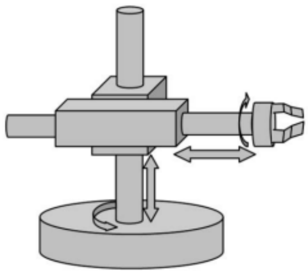

Fig. 1. Industrial robot manipulator M20P

The industrial robot manipulator M20P has two translational and two rotational kinematic pairs. The elements of manipulator are numbered starting with fixed element - the base with number of zero. The motion of clamp-unclamp for the gripping device is not considered. The kinematic scheme for industrial robot manipulator is shown in Figure 2 and consists of four moving parts. We introduce the relative coordinate system associated with elements, with the origin at the points: O_1, O_2, O_3, O_4…,The initial coordinate system O0 we correlate with the fixed element. For generalized coordinates we accept: the angle of rotation around the rack q- 1 , lifting height q- 2 , the length of arms q- 3 , the angle of gripping device q- 4 . The system of dynamic equations of industrial robot manipulator is obtained. We apply the matrix method and dynamic Lagrange equations in matrix form to produce the equations of motion [5-7]. The transition from the coordinate system O_0 to O_1 occurs by rotation around z axis at an angle q1.

The transition from the coordinate system O_1 to O_2 occurs by rotation around z axis at an angle π , by displacement along z axis on q2 and by rotation around x axis at an angle $\pi /2$. The transition from the coordinate system O_2 to O_3 occurs by displacement along z axis on q3 . The transition from the coordinate system O_3 to O_4 occurs by rotation around z axis at an angle q4, by displacement along z axis on s and by rotation around x axis at an angle $-\pi /2$. We introduce the radius vector of points O_i , (i=1,2,3,4) in the i-local coordinate system: Ri = [xi, yi, zi, 1] ^{T} the communication of radius vectors in the coordinate system i-1 and I by means the transition matrix A_{i-1}: $R_{i-1} = A_{i-1,i}R_i$,. The transition matrix from point O_0 to point O_1 is of the form:

$$A_{01} = \begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The matrices of the transition from point O_1 to point O_2 and of the transition from point O_2 to point O_3 are of the form:

$$A_{12} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transition matrix from point O_3 to point O_4 is of the form:

$$A_{34} = \begin{pmatrix} \cos(q_4) & 0 & -\sin(q_4) & 0 \\ \sin(q_4) & 0 & \cos(q_4) & 0 \\ 0 & -1 & 1 & s \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transition matrix from point O_0 to point O_2 is defined as the product of matrices: $A_{02} = A_{01}A_{12}$

$$A_{02} = \begin{pmatrix} -\cos(q_1) & 0 & -\sin(q_1) & 0 \\ -\sin(q_1) & 0 & \cos(q_1) & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transition matrix from point O0 to point O3 is obtained as the product of matrices: A03 = $A_{01}A_{12}A_{23}$,

$$A_{04} = \begin{pmatrix} -\cos(q_1)\cos(q_4) & \sin(q_1) & \cos(q_1)\sin(q_4) & 0 \\ -\cos(q_4)\sin(q_1) & -\cos(q_1) & \sin(q_1)\cos(q_4) & 0 \\ sin(q_4) & 0 & \cos(q_4) & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The origin of coordinates O_4 associated with gripping device in the fixed coordinate system O_0 associated with a base of rack is defined by coordinates:

$$x_{04} = - q_3\sin(q_1) – a \sin(q_1),$$
$$y_{04} = q_3\cos(q_1) + s \cos(q_1),$$
$$z_{04} = q_2$$

We denote the coordinates of the gripper fingers in a coordinate system O_4 as (x_4, y_4, z_4), then in a fixed coordinate system O_0 the coordinates of the gripper. Fingers can be represented in the form of:

$$x_{04} = - q_3(\sin q_1) – x_4\cos(q_1)\cos(q_4) + z_4\cos(q_1)\sin(q_4) – s\sin(q_1) + y_4\sin(q_1)$$
$$y_{04} = q_3\cos(q_1) – x_4\sin(q_1)\cos(q_4) + z_4 \sin(q_1)\sin(q_4) + s\cos(q_1) – y_4\cos(q_1)$$
$$z_{04} = q_2 + x_4\sin(q_4) + z\cos(q_4)$$

CONCLUSION

The mathematical model of industrial robot manipulator with four degrees of freedom is developed. The robot manipulator is designed to create automated systems for service devices, for removing and installing the equipment, change details and tools. For creation of the equations of robot manipulator we used the method of matrices and matrix Lagrange equations of the second kind. The mathematical model is presented nonlinear system of four ordinary differential equations of second order. The analytical solution of the nonlinear system is obtained by polynomial transformations method. The kinematic structure and dynamic characteristics of robot manipulator are defined. The power of actuators and generalized forces generated by actuators for moving the gripper at a given point in space are found.

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