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EXPERIMENTAL AND THEORETICAL INVESTIGATION OF INTERACTION EFFECT ON ENERGY ABSORPTION OF BI-TUBULAR STRUCTURES UNDER QUASI-STATIC AXIAL CRUSHING

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ABSTRACT

This work reveals the interaction effect on energy absorption of bi-tubular, a combination of outer circular tube with inner polygonal tube structures under quasi-static axial crushing. First, the experimental mean crushing force of single tubes was compared with theoretical prediction and it witness good agreement between them. In bi-tubular arrangement, the inscribed diameter of inner polygonal tubes plays an important role in improving the mean crushing force by interaction effect. The mean crushing force of bi-tubular specimens D3T, D3S and D3H have 35, 26, 10.1 percentage higher than the summation of their individual single tubes that is SC with S3T or S3S or S3H. **Keywords:** bi-tubular structures, quasi-static axial crushing, interaction effect, mean crush force, specific energy absorption.

Nomenclature

D	-	Diameter of the tube, m
$M_o = \sigma_o h^2/4$	-	Full plastic bending moment of tube wall per unit length, N
m	-	Mass of the thin walled structure, kg
F _{mean}	-	Mean crushing force, N
F _{Peak}	-	Peak crushing force, N
С	-	Side of the polygon, m
h or t	-	Thickness of the tube, m
δ	-	Crushing distance or Crushing deflection, m

σ₀	-	Flow stress, N/m ²
σ_{u}	-	Ultimate tensile stress, N/m ²
σ _y	-	Yield stress, N/m ²

1. Introduction

During collision, energy absorption devices are responsible to reduce the loss of life and damage of vehicle critical parts. These devices can absorb maximum kinetic energy by permanent plastic deformation and transferred minimum amount of force to occupants. From the different types of energy absorbers, the metallic thin-walled tubes are considered one of the most efficient energy absorbing devices for their ease manufacture, high strength and low cost [1, 2]. Over the past few decades many researchers studied about crashworthiness of thin walled cylinders and have submitted in experimental, analytical and numerical results with different cross sections, materials, tubes filled with foam and multi cell thin walled structures [3, 4].

The analysis of axial crushing on thin walled cylinders was pioneered by Alexander [5] and the work yield an excellent theoretical model to access average crushing force for axisymmetric fold pattern. Subsequently, Abramowicz W and Jones [6] analysed the different types of crushing modes and took more efforts to improve the previous theoretical models by number of experiments on the concertina or ring mode for circular cylinders. Wu Hong et al. [7] studied about collapse mode and energy absorption of triangular tubes under quasi-static axial compression experiments. Based on folding elements, they proposed four different theoretical models to predict the mean crushing force. Abramowicz et al. [8] studied about the effect of corner element of an arbitrary angle on energy absorption by using rigorous methods of structural plasticity with finite deformations and rotations. Based on this analysis they developed mathematical models to predict the mean crushing force of multi-corner columns with an even number of corners such as square, hexagonal, and rhomboidal thin-walled columns.

Alavi Nia et al [9, 10] and Velmurugan et al [11, 12] explored different configurations in single material and showed that geometric parameter in a conventional way to improve the crashworthiness of thin walled structures.

When using foam, honeycomb or sawdust as filler materials, the energy absorption capacity of the thin-walled structures was found to be increased by interaction between them. Hanssen et al. [13-15] carried out complete studies on aluminium columns with aluminium foam filler and identified a strong foundation for the subsequent theoretical analysis and practical application. In recent years, foam filled tubes with various cross sections such as circular [13], square [14-16], polygonal [17] and elliptical [18] have been carried out extensively.

The effect of length difference on crashworthiness performance of bi-tubal structures was reported by Sharifi et al. [19], Vinayagar et al. [20] and Kashani et al. [21]. Recently, Vinayagar et al. [22], Abbas Rahi [23], Muhammad Kamran et al. [24] were reported the influence of geometry on the energy absorption of bi-tubes under axial compression.

Though the crushing of bi-tubular systems are very recent topics in crashworthiness analysis, the discussion about interaction effect with quantitative results are very minimum. The energy absorption capabilities of single tubes with various sections were studied experimentally and experimental results were compared with theoretical predictions [22]. Due to interaction effect, the bi-tubular combination has higher mean crushing force, when compared with the summation of mean

crushing force of individual single tubes and the percentage of improvement in mean crushing force also reported.

2. Definition of crashworthiness indicators

With the purpose of better understanding about crashworthiness of thin-walled structures, some indicators like Total Energy Absorption (TEA), Specific Energy Absorption (SEA), and Mean crush force (F_{mean}) are described with brief explanation.

Total Energy Absorption

Total Energy Absorption expresses the energy absorption capacity of specimens. It can be determined by integration of the axial crushing force versus displacement.

$$TEA = \int_{0}^{\delta} F(\delta) d\delta$$
 (1)

where $F(\delta)$ is the instantaneous crushing force with a function of the crushing distance δ . From experiments or numerical modeling, the instantaneous crushing force can be obtained.

Specific Energy Absorption

Specific Energy Absorption is defined as energy absorbed per unit mass of the thin-walled member. It is used to compare the energy absorption capacity of structures with different mass which is given by SEA = TEA/m

(2)

(3)

Mean Crush Force

The average crush force is also one of the response parameter for the energy absorption capability of a structure, which is calculated as,

 $F_{mean} = TEA/\delta$

where TEA is energy absorbed during collapse with displacement δ .

3. Specimen Preparation

Since the desirable tube sections were not available in the market, the specimens were prepared in sheet metal shop from 0.478 mm thick SS304 stainless steel sheet. The stainless steel sheet was cut into the required dimensions and joined as tube by TIG welding process. Stainless steel 304 is considered for its good formability and strength. Mechanical properties of this sheet metal were presented in Table 1.

Table 1 Mechanical prop	perties of 304-stainless steel sheet

Property	Ultimate Tensile Strength (MPa)	Yield Stress (MPa)	% of Elongation
Value	531.2	223.6	41



Fig.1 Section geometry and dimensions of single tube specimens

(All dimensions are in mm)

Single tubes of various polygonal sections such as triangle, square and hexagon were fabricated according to three levels of inscribed diameter of polygonal section such as 60 mm, 70 mm and 80 mm. The edge length of various polygonal sections are derived from the inscribed diameter of the polygonal section as shown in Figure 1 and tabulated in Table 2. Length of all the single tubes is 135 mm.

Sl. No.	Code	Inscribed diameter of the polygonal section (mm)	Section of the tube	Side/Edge length of Polygon (mm)
	S1T	60	Triangle	52
	S1S	60	Square	42
	S1H	60	Hexagon	30

S2T	70	Triangle	61
S2S	70	Square	49
S2H	70	Hexagon	35
S3T	80	Triangle	69
S3S	80	Square	57
S3H	80	Hexagon	40
SC	90	Cylinder	Ø 90

3.1 Coding System of Single Tubes

The prepared single tube specimens were coded for the purpose of easy identification, evaluation and performance comparison. In coding of single polygonal tubes, the first alphabet 'S' indicates single tube and the second numerical number 1, 2 and 3 indicates the level of inscribed diameter of the polygonal section i.e., 60 mm, 70 mm and 80 mm respectively then the third alphabet indicates section of tube i.e., T for Triangular, S for Square and H for Hexagonal section. The coding method of single tubes is noted in Table 3.

Coding Order	Code	Description
First alphabet	S Single tube	
	1	Inscribed diameter of polygonal section is 60 mm
Second numerical number	2	Inscribed diameter of polygonal section is 70 mm
	3	Inscribed diameter of polygonal section is 80 mm
	Т	Polygonal section of tube is triangle
Third alphabet	S	Polygonal section of tube is square
	Н	Polygonal section of tube is hexagon

Table 3 Coding system of single tubes

For example, S2H means single tube with 70 mm (second level) inscribed diameter of the polygonal section and its section was hexagon. Then the circular cylinder was fabricated with the diameter of 90 mm. The single cylinder was coded as SC, here also 'S' stands for single and 'C' stands for cylinder. For consistency of output in each type of geometry, three samples were fabricated.

3.2 Experimental process

For this quasi static axial compression analysis, FSA make Model TUF-CN 1000 servo controlled universal testing machine (Figure 2) was used. The samples were crushed at a loading rate of 10 mm/min as previous study [22]. Here also, the specimens were crushed up to 100 mm at the room temperature of 25°C.

4. Results and discussion

Load-displacement curves of the single tube specimens are obtained from quasi static tests. The important crashworthiness parameters such as mean crushing force, total energy absorption and specific energy absorption were calculated from the load-displacement curves of the single tube specimens and recorded in Table 4.



Fig.2 Computerized - Servo controlled Universal Testing Machine

Specimen Code	Mass (kg)	F _{mean} (kN)	F _{Peak} (kN)	TEA (J)	SEA (kJ/kg)	CFE (%)
SC	0.140	8.18	16.08	820.7	5.862	50.87
S1T	0.080	2.81	5.57	281.3	3.516	50.45
S1S	0.083	4.45	8.11	458.8	5.528	54.87
S1H	0.090	6.10	11.59	611.1	6.790	52.63
S2T	0.095	3.11	5.78	318.9	3.357	53.81
S2S	0.100	4.62	8.54	475.6	4.756	54.10
S2H	0.103	6.96	12.56	698.4	6.780	55.41
S3T	0.105	3.25	5.95	325.8	3.103	54.62
S3S	0.115	4.88	8.96	491.8	4.277	54.46
S3H	0.120	7.55	15.18	758.6	6.322	49.74

 Table 4 Experimental results of single tubes

Concertina mode is developed in specimen SC, in-extensional mode is developed in single polygonal tubes and diamond mode is observed in all bi-tubular specimens. Based on experimental results, a detailed comparison report of the crashworthiness performance between various polygonal sections in three levels of inscribed diameter of single tubes was presented.

4.1 Effect of Inscribed Diameter on Mean Crushing Force of Single Tubes

On crashworthiness, the effect of the inscribed diameter of polygonal section of single tubes is studied. The edge length of polygon is increased with the inscribed diameter of polygonal section from 60 mm, 70 mm and 80 mm. When the area of polygonal sections is increased the stability of tubes also increased. This is the main reason for improvement of energy absorption capability. Mean crushing force of tested sections for various inscribed diameter of polygonal sections is compared in Figure 3. It can be understood from this diagram for the same section of single tube, mean crushing force increased with increasing of the inscribed diameter of polygonal section. As in Table 4, the total energy absorption of the thin walled tubular structures with inscribed diameter of polygonal section of 60 mm, 70 mm and 80 mm is the least, medium and the most respectively.



Fig. 3 Effect of inscribed diameter of polygonal section on mean crushing force of single tubes

4.2 Effect of Polygonal Section of Single Tube

Thin-walled structures are used to dissipate the impact energy through the process of plastic deformation. In fact, the studies [9, 10] have shown that the amount of absorbed energy increased with the number of sides of a section. The reason for this is the number of plastic hinges and folds are increased with number of sides of polygon. Figure 4 shows that the comparisons of mean crushing force between various sections of tubes for a particular inscribed diameter of the polygon. Figure 4 is also pointed out the mean crushing force increased by increasing the number of edges or corners of the section of any of inscribed diameter of the polygonal section.



Fig.4 Effect of polygonal section on mean crushing force of single tubes

From Table 4, TEA of hexagonal tube was greater than the square and the square was greater than the triangle, i.e., in anyone of inscribed diameter of the polygonal section, the greater the number of sides, the absorbed energy is more. The hexagonal tubes have more SEA than the other sections as presented in Table 4 and is shown in Figure 5.

4.3 Theoretical predictions

Comparison of experimental results with theoretical predictions was must to validate the experiments. To predict the mean crushing force of single tubes, there are several theoretical models have been addressed by many researchers for concertina and diamond mode of various sectional tubes such as circular, triangle, square and hexagon.



Fig.5 Effect of polygonal section on SEA of single tubes

Theoretical model of circular cylinder

The important expression of mean crushing force for concertina mode of circular cylinder is derived by W.

Abramowicz and N. Jones [6] in 1984.

 $F_{mean} = \frac{20.79(\frac{D}{t})^{0.5} + 11.9}{0.86 - 0.568(\frac{t}{D})^{0.5}} M_o$

Theoretical model of Triangular Tube

For triangular tube Wu Hong et al. [7] presented the following model for the mean crushing force which is,

$$F_{mean} = 26.66 M_o \left(\frac{C}{t}\right)^{\frac{1}{3}}$$
⁽⁵⁾

Theoretical model of square Tube

The following classic theoretical model for square tube under axial compression was proposed by Abramowicz and Wierzbicki [8],

$$F_{mean} = 12.16\sigma_o C^{0.37} t^{1.63} \tag{6}$$

Most of experimental studies were validated based on this theoretical model.

Theoretical model of Hexagonal Tube

The theoretical expression presented by Abramowicz and Wierzbicki [8] for hexagonal column is approximated by the following functions,

$$F_{mean} = 20.2\sigma_o C^{0.4} t^{1.6}$$

Determination of Flow Stress

Actually flow stress is not a constant value, so determination of the flow stress is very important. During loading the strains for different collapse modes and in different regions of tubes are different. Therefore different expressions were employed by the researchers to calculate the flow stress. In this present work, the average value of yield stress and ultimate tensile stress is employed as flow stress for all of the thin walled structures.

Table 5 Experimental results Vs theoretical Predictions of single tubes

Specimon	Mean Crushing I	Porcontago	
code	Experimental Result	Theoretical Prediction	of deviation
SC	8.18	7.8	4.8
S1T	2.81	2.7	4.0
S2T	3.11	2.9	7.2
S3T	3.25	3.01	7.9
S1S	4.45	4.3	3.5
S2S	4.62	4.53	2.0
S3S	4.88	4.74	3.0
S1H	6.1	6.5	6.6
S2H	6.96	6.8	2.4
S3H	7.55	7.3	3.4

(4)

(7)

Based on the above mathematical equations (4, 5, 6, 7) theoretical mean crushing force was calculated for all section of tubes. Then the values of experimental mean crushing forces of single tubes for circular, triangular, square and hexagonal sections are compared with theoretically predicted values in Table 5. From the Table 5, the percentage of deviation is very less and it can be understood that the theoretical predictions are found to be in a good agreement with the experimental results.

4.4 Comparison between single and bi-tubular structures

Experimental results of crashworthiness indicators for bi-tubular structures are taken from the previous work [22] and tabulated in Table 6. Table 6 shows, the mean crushing force, total energy absorption and specific energy absorption of the bi-tubes were greater than the single tubes.

Code	Mass (kg)	F _{Peak} (kN)	F _{mean} (kN)	TEA (J)	SEA (kJ/kg)
D1T	0.220	21.80	10.76	1081.8	4.917
D1S	0.223	24.12	12.59	1265.7	5.676
D1H	0.230	29.13	14.57	1459.5	6.346
D2T	0.235	22.85	12.11	1227.7	5.224
D2S	0.240	26.01	14.36	1475.5	6.148
D2H	0.243	31.17	16.28	1632.9	6.718
D3T	0.245	24.50	15.40	1541.2	6.291
D3S	0.255	26.83	16.43	1648.5	6.465
D3H	0.260	33.87	17.42	1747.6	6.722

Table 6	Experimental	results	of	bi-tubes
	Lipermentur	I COULCO	•••	

4.5 Influence of interaction effect in bi-tubes

Each of the single polygonal tubes and single circular cylinder was compressed separately and the summation of the mean crushing force was compared with the bi-tubular specimens in Table 7.

In D1T, D1S and D1H the clearance between the tubes is more. While crushing, the contact between the tubes is very less and both tubes are crushed almost separately. So the mean crushing force and the absorbed energy of D1T, D1S and D1H bi-tubular specimens are approximately equal to the summation of their single tubes that is SC with S1T or S1S or S1H.

Table / Improvement of mean crushing force in bi-tubes							
Specin	nen code	Experimental mean crushing	Improvement in				
Single tubes	Equivalent bi- tube	Summation of single tubes	Summation of single tubes Bi-tube				
SC+S1T	D1T	8.18+2.81=10.99	10.76	2.1			
SC+S2T	D2T	8.18+3.11=11.29	12.11	7.3			
SC+S3T	D3T	8.18+3.25=11.43	15.4	35			
SC+S1S	D1S	8.18+4.45=12.63	12.59	0.32			
SC+S2S	D2S	8.18+4.62=12.8	14.36	12.3			
SC+S3S	D3S	8.18+4.88=13.06	16.43	26			
SC+S1H	D1H	8.18+6.1=14.28	14.57	2			
SC+S2H	D2H	8.18+6.96=15.14	16.28	7.5			
SC+S3H	D3H	8.18+7.55=16.73	17.42	10.1			

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In D2T, D2S and D2H the clearance between the tubes is less compared to D1T, D1S and D1H. While crushing the contact between the tubes can cause the growth in mean crushing load and absorbed energy. So the mean crushing force of D2T, D2S and D2H bi-tubular specimens are 7.3, 12.3, 7.5 percentage higher than the summation of their single tubes that is SC with S2T or S2S or S2H.



Fig. 6 Effect of interaction on mean crushing force

Like that, in D3T, D3S and D3H the clearance between the tubes is very less compared to D1T, D1S and D1H. While crushing the contact between the two tubes becomes tighter, it can cause the growth in mean crushing load. The mean crushing force of D3T, D3S and D3H bi-tubular specimens are 35, 26, 10.1 percentage higher than the summation of their single tubes that is SC with S3T or S3S or S3H. Table 7 and Figure 6 are clearly shown that the improvements of mean crushing force of bi-tubular specimens are increased with inscribed diameter of inner tube.

5. Conclusions

In this research paper, the crashworthiness behavior of single thin-walled tubes with circular, triangular, square and hexagonal sections was studied under quasi-static axial compression. From this experimental study the following conclusion can be drawn:

1. In single tubes the Total Energy Absorption (TEA) and mean crushing force (F_{mean}) are increased with increasing the inscribed diameter of polygonal section. F_{mean} and Specific Energy Absorption (SEA) are increased with increasing the number of edges or corners of the section for any one of inscribed diameter of polygon.

2. Experimental mean crushing force of single tubes was compared with theoretical values and found that they have good consistency between them. Bi-tubular structures had greater SAE than the single tubes.

3. Effect of interaction or clearance between the tubes in bi-tubular arrangement was analyzed in detail. The mean crushing force of D1T, D1S and D1H bi-tubular specimens are approximately equal to the summation of their single tubes that is SC with S1T or S1S or S1H.

4. In the second and third level of inscribed diameter of inner tubes, the clearance between the tubes is minimum. Less clearance makes more interaction between the tubes and it causes more energy absorption in bi-tubular arrangement. The mean crushing force for D3T was 35% greater than the summation of their single tubes that is SC with S3T. From this analysis the interaction effect between the tubes are proved experimentally.

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