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M/M/1/K/N INTERDEPENDENT RETRIAL QUEUEING MODEL WITH CONTROLLABLE ARRIVAL RATES AND CUSTOMERS ABANDONMENT

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Abstract
In this paper, a finite capacity single server finite population interdependent retrial queueing model with controllable arrival rates and customers abandonment is considered. The steady state solutions and the system characteristics are derived and analyzed for this model. Some particular cases of the model have been discussed. This model may be of great importance to the business facing the serious problem of customer impatience. Numerical results are given for better understanding and relevant conclusion is presented

Keywords: retrial queue; finite source; customer abandonment; interdependent primary arrival and service processes; finite capacity.

1. Introduction

Queueing systems in which primary customers who find all the servers and waiting positions (if any) occupied may retry for service after a long period of time are called retrial queues. Between retrials a customer is said to be in orbit. Abandonment happens when a subscriber's call becomes rejected and the subscriber gets impatient and gives up after a certain time without getting service. The model studied in this paper not only takes into account retrials due to congestion but also considers the effects of customers abandonment

In this paper, the M/M/1/K/N interdependent retrial queueing model with controllable arrival rates and customers abandonment is considered. In section 2, the description of the model is given stating the relevant postulates. In section 3, the steady state equations are obtained. In section 4, the characteristics of the model are derived. In section 5, numerical results are calculated.

2. Description of The Model

Consider a single server finite capacity finite source retrial queueing system in which primary customers arrive according to the Poisson flow of rate λ_0 and λ_1, service times are exponentially distributed with rate μ. If a primary customer finds some server free, he instantly occupies it and leaves the system after service. Otherwise, if the server is busy, at the time of arrival of a primary call then with probability $H_1 \geq 0$ the arriving customer enters an orbit and repeats his demand after an exponential time with rate θ. Thus the Poisson flow of repeated call follow the retrial policy where the repetition times of each customer is assumed to be independent and exponentially distributed.

If an incoming repeated call finds the line free, it is served and leaves the system after service, while the source which produced this repeated call disappears. Otherwise, if the server is occupied at the time of a repeated call arrival with probability $(1-H_2)$ the source leaves the system without service.
It is assumed that the primary arrival process $[X_1(t)]$ and the service process $[X_2(t)]$ of the systems are correlated and follow a bivariate Poisson process given by

$P(X_1=x_1, X_2=x_2;t)$

$$= e^{-(\lambda_i + \mu - \varepsilon)t} \frac{\sum_{j=0}^{\min(x_1 x_2)} \varepsilon(t)^j \ (\lambda_i-\varepsilon)t^{x_1-j} \ (\mu-\varepsilon)t^{x_2-j}}{j!\,(x_1-j)!\,(x_2-j)!}$$

$x_1, x_2 = 0,1,2,\ldots\ldots,\lambda_i, \mu < 0,\ i = 0,1;$

with parameters λ_0, λ_1, μ_n and ε as mean faster rate of primary arrivals, mean slower rate of primary arrivals, mean service rate and mean dependence rate (covariance between the primary arrival and service processes) respectively.

3. Steady State Equation

Let $P_{0,n,0}$ denote the steady state probability that there are n customers in the queue when the system is in the faster rate of primary arrivals and the server is idle.
Let $P_{0,n,1}$ denote the steady state probability that there are n customers in the queue when the system is in the slower rate of primary arrivals and the server is idle.
Let $P_{1,n,0}$ denote the steady state probability that there are n customers in the queue when the system is in the faster rate of primary arrivals and the server is busy.
Let $P_{1,n,1}$ denote the steady state probability that there are n customers in the queue when the system is in the slower rate of primary arrivals and the server is busy.
We observe that only $P_{0,n,0}$ and $P_{1,n,0}$ exists when n=0,1,2, ……., r-1,r; $P_{0,n,0}$, $P_{1,n,0}$, $P_{0,n,1}$ and $P_{1,n,1}$ exist when n=r+1,r+2,……, R-2,R-1; $P_{0,n,1}$ and $P_{1,n,1}$ exists when n=R,R+1,….. , K. Further $P_{0,n,0}= P_{1,n,0} = P_{0,n,1} = P_{1,n,1} = 0$ if n>K.
The steady state equations are

$$-N(\lambda_0-\varepsilon)P_{0,0,} + (\mu-\varepsilon)P_{1,0,0}=0 \quad (1)$$

$$-[(N-1)H_1(\lambda_0-\varepsilon)+(\mu-\varepsilon)]P_{1,0,0} + N(\lambda_0-\varepsilon)P_{0,0,0} + \theta P_{0,1,0} + \theta(1-H_2) P_{1,1,0} = 0 \quad (2)$$

$$-[(N-n)(\lambda_0-\varepsilon)+n\theta]P_{0,n,} + (\mu-\varepsilon)P_{1,n,0}=0 \quad (3)$$

$$-[(N-n-1)H_1(\lambda_0-\varepsilon)+(\mu-\varepsilon)+n\theta(1-H_2)] P_{1,n,0} +[(N-n)(\lambda_0-\varepsilon)] P_{0,n,0}$$
$$+ [(N-n)H_1(\lambda_0-\varepsilon)] P_{1,n-1,0} + (n+1)\theta P_{0,n+1,0}+ [(n+1)\theta(1-H_2)] P_{1,n+1,0}=0,$$
$$n=1,2,3,\ldots..,r-1 \quad (4)$$

$$-[(N-r)(\lambda_0-\varepsilon)+r\theta]P_{0,r,0} + (\mu-\varepsilon)P_{1,r,0}=0 \quad (5)$$

$$-[(N-r-1)H_1(\lambda_0-\varepsilon)+(\mu-\varepsilon)+r\theta(1-H_2)] P_{1,r,0} +[(N-r)(\lambda_0-\varepsilon)] P_{0,r,0} + [(N-r)H_1(\lambda_0-\varepsilon)] P_{1,r-1,0}$$
$$+ (r+1)\theta P_{0,r+1,0}+[(r+1)\theta(1-H_2)] P_{1,r+1,0}+(r+1)\theta P_{0,r+1,1}+ [(r+1)\theta(1-H_2)] P_{1,r+1,1}=0 \quad (6)$$

$$-[(N-n)(\lambda_0-\varepsilon)+n\theta]P_{0,n,} + (\mu-\varepsilon)P_{1,n,0}=0 \quad (7)$$

$$-[(N-n-1)H_1(\lambda_0-\varepsilon)+(\mu-\varepsilon)+n\theta(1-H_2)] P_{1,n,0} +[(N-n)(\lambda_0-\varepsilon)] P_{0,n,0} + [(N-n)H_1(\lambda_0-\varepsilon)] P_{1,n-1,0}$$
$$+ (n+1)\theta P_{0,n+1,0}+ [(n+1)\theta(1-H_2)] P_{1,n+1,0}=0,$$
$$n= r+1, r+2,\ldots\ldots,R-2 \quad (8)$$

$$-[(N-R+1)(\lambda_0-\varepsilon)+(R-1)\theta]P_{0,R-1,}+ (\mu-\varepsilon)P_{1,R-1,0}=0 \quad (9)$$

$-[(N-R)H_1(\lambda_0-\varepsilon)+(\mu-\varepsilon)+(R-1)\theta(1-H_2)]\,P_{1,R-1,0}+[(N-R+1)(\lambda_0-\varepsilon)]\,P_{0,R-1,0}$

$+[(N-R+1)H_1(\lambda_0-\varepsilon)]P_{1,R-2,0}=0$ (10)

$-[(N-r-1)(\lambda_1-\varepsilon)+(r+1)\theta]P_{0,r+1,1}+(\mu-\varepsilon)P_{1,r+1,1}=0$ (11)

$-[(N-r-2)H_1(\lambda_1-\varepsilon)+(\mu-\varepsilon)+(r+1)\theta(1-H_2)]\,P_{1,r+1,1}+[(N-r-1)(\lambda_1-\varepsilon)]\,P_{0,r+1,1}+$

$[(N-n)H_1(\lambda_1-\varepsilon)]P_{0,r+1,1}+(r+2)\theta\,P_{0,r+2,1}+[(r+2)\theta(1-H_2)]\,P_{1,r+2,1}=0$ (12)

$-[(N-n)(\lambda_1-\varepsilon)+n\theta]P_{0,n,1}+(\mu-\varepsilon)P_{1,n,1}=0$ (13)

$-[(N-n-1)H_1(\lambda_1-\varepsilon)+(\mu-\varepsilon)+n\theta(1-H_2)]\,P_{1,n,1}+[(N-n)(\lambda_1-\varepsilon)]\,P_{0,n,1}+$

$[(N-n)H_1(\lambda_1-\varepsilon)]\,P_{1,n-1,1}+(n+1)\theta\,P_{0,n+1,1}+[(n+1)\theta(1-H_2)]\,P_{1,n+1,1}=0,$

$n = r+2, r+3, \ldots\ldots. R-1$ (14)

$-[(N-R)(\lambda_1-\varepsilon)+R\theta]P_{0,R,1}+(\mu-\varepsilon)P_{1,R,1}=0$ (15)

$-[(N-R-1)H_1(\lambda_1-\varepsilon)+(\mu-\varepsilon)+R\theta(1-H_2)]\,P_{1,R,1}+[(N-R)(\lambda_1-\varepsilon)]\,P_{0,R,1}$

$+[(N-R)H_1(\lambda_1-\varepsilon)]\,P_{1,R-1,1}+[(N-R)H_1(\lambda_0-\varepsilon)]\,P_{1,R-1,0}+(R+1)\theta\,P_{0,R+1,1}$

$+[(R+1)\theta(1-H_2)]P_{1,R+1,1}=0$ (16)

$-[(N-n)(\lambda_1-\varepsilon)+n\theta]P_{0,n,1}+(\mu-\varepsilon)P_{1,n,1}=0$ (17)

$-[(N-n-1)H_1(\lambda_1-\varepsilon)+(\mu-\varepsilon)+n\theta(1-H_2)+(n-1)]\,P_{1,n,1}+[(N-n)(\lambda_1-\varepsilon)]\,P_{0,n,1}+$

$[(N-n)H_1(\lambda_1-\varepsilon)]\,P_{1,n-1,1}+(n+1)\theta\,P_{0,n+1,1}+[(n+1)\theta(1-H_2)]\,P_{1,n+1,1}=0,$

$n = R+1, R+2, \ldots\ldots\ldots K-1$ (18)

$-[(N-K)(\lambda_1-\varepsilon)+K\theta]P_{0,K,1}+(\mu-\varepsilon)P_{1,K,1}=0$ (19)

$-(\mu-\varepsilon)P_{1,K,1}+[(N-K)H_1(\lambda_1-\varepsilon)]P_{1,K-1,1}+[(N-K)(\lambda_1-\varepsilon)]\,P_{0,K,1}=0$ (20)

Write $T_1=[H_1(\lambda_0-\varepsilon)]$ and $T_2=[H_1(\lambda_1-\varepsilon)]$

From (1) to (4) we get,

$$P_{0,n,0}=\frac{(N-1)_n T_1{}^n \prod_{i=0}^{n-1}[(N-i)(\lambda_0-\varepsilon)+i\theta]}{\prod_{i=1}^{n} i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_0-\varepsilon)+i\theta]}P_{0,0,0} \quad 1\leq n\leq r \qquad (21)$$

$$P_{1,n,0}=\frac{(N-1)_n T_1{}^n}{\mu-\varepsilon}\prod_{i=1}^{n}\frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_0-\varepsilon)+i\theta]} \quad P_{0,0,0} \qquad (22)$$

From (5) to (8) we get,

$$P_{0,n,0}=\frac{(N-1)_n T_1{}^n \prod_{i=0}^{n-1}[(N-i)(\lambda_0-\varepsilon)+i\theta]}{\prod_{i=1}^{n} i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_0-\varepsilon)+i\theta]}P_{0,0,0}$$

$$-\left\{\frac{A_1}{A_2}\left(\left[\sum_{m-r}^{n-2}(N-m-)T_1^{\;n-1-m}\prod_{i=m+1}^{n}\frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_0-\varepsilon)+i\theta]}\right]+\right)\right\}\quad P_{0,r+1,1} \tag{23}$$

$$P_{1,n,0}=\frac{(N-1)_nT_1^{\;n}}{\mu-\varepsilon}\prod_{i=1}^{n}\frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_0-\varepsilon)+i\theta]}\quad P_{0,0,0}$$

$$-\left\{\frac{A_1}{A_2(\mu-\varepsilon)}\left(\left[\sum_{m-r}^{n-1}(N-m-2)T_1^{\;n-1-m}\prod_{i=m+1}^{n}\frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_0-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1}$$

$n = r+1, r+2, \ldots\ldots, R-1$ (24)

where

$A_1 = (r+1)\theta(\mu-\varepsilon)+[(r+1)\theta(1-H_2)][(N-r-1)(\lambda_1-\varepsilon)+(r+1)\theta]$

$A_2 = n\theta(\mu-\varepsilon)+[n\theta(1-H_2)][(N-n)(\lambda_0-\varepsilon)+n\theta]$

From (9) to (10) we get,

$P_{0,r+1,1} = \frac{A_3}{A_4}p_{0,0,0}$

Where $A_3=\frac{(N-1)_RT_1^{\;R}\prod_{i=0}^{R-1}[(N-i)(\lambda_0-\varepsilon)+i\theta]}{\prod_{i=1}^{R-1}i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_0-\varepsilon)+i\theta]}P_{0,0,0}$

$$A_4=-\left\{A_1\left(\left[\sum_{m-r}^{R-2}(N-m-2)T_1^{\;n-1-m}\prod_{i=m+1}^{R-1}\frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_0-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1} \tag{25}$$

From (11) to (14), we recursively derive,

$$P_{0,n,1}=\left\{\frac{A_1}{A_5}\left(\left[\sum_{m-r}^{n-2}(N-m-2)T_2^{\;n-1-m}\prod_{i=m+1}^{n-1}\frac{[(N-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_0-\varepsilon)+i\theta]}\right]+1\right)\right\}\quad P_{0,r+1,1} \tag{26}$$

$$P_{1,n,1}=\left\{\frac{A_1}{\mu-\varepsilon}\left(\left[\sum_{m-r}^{n-1}(N-m-2)T_2^{\;n-1-m}\prod_{i=m+1}^{n}\frac{[(N-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_1-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1}$$

$n = r+1, r+2, \ldots\ldots R-1, R$ (27)

where

$A_5 = n\theta(\mu-\varepsilon) + [n\theta(1-H_2)][(N-n)(\lambda_1-\varepsilon)+n\theta]$

A_1 is given by (23) and $P_{0,r+1,1}$ is given by (25)

From (15) to (20) we recursively derive,

$$P_{0,n,1} = \left\{\frac{A_1}{A_5}\left(\left[\sum_{m-r}^{R-2}(N-m-2)T_2^{\,n-1-m}\prod_{i=m+1}^{n-1}\frac{[(N-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_0-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1} \quad (28)$$

$$P_{1,n,1} = \left\{\frac{A_1}{\mu-\varepsilon}\left(\left[\sum_{m-r}^{R-1}(N-m-2)T_2^{\,n-1-m}\prod_{i=m+1}^{n}\frac{[(N-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)][(N-i)(\lambda_1-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1}$$

n=R+1, R+2,…K-1,K (29)

where A_1, A_5 and $P_{0,r+1,1}$ are given by (23), (25), (26).

Thus from (21) to (29), we find that all the steady state probabilities are expressed in terms of $P_{0,0,0}$.

4. Characteristics of The Model

- The probability P(0)that the system is in faster rate of primary arrivals with the server idle and busy.
- The probability P(1) that the system is in slower rate of primary arrivals with the server idle and busy.
- The probability $P_{0,0,0}$ that the system is empty.
- The expected number of customers in the system Ls_0, when the system is in faster rate of primary arrivals with the server idle and busy.
- The expected number of customers in the system Ls_1, when the system is in slower rate of primary arrivals with the server idle and busy.

The probability that the system is in faster rate of primary arrivals is

$$P(0) = \left[\sum_{n=0}^{r}P_{0,n,0} + \sum_{n=r+1}^{R-1}P_{0,n,0}\right] + \left[\sum_{n=0}^{r}P_{1,n,0} + \sum_{n=r+1}^{R-1}P_{1,n,0}\right] \quad (30)$$

The probability that the system is in slower rate of primary arrivals is,

$$P(1) = \left[\sum_{n=r+1}^{R}P_{0,n,1} + \sum_{n=R+1}^{K}P_{0,n,1}\right] + \left[\sum_{n=r+1}^{R}P_{1,n,1} + \sum_{n=R+1}^{K}P_{1,n,1}\right] \quad (31)$$

The probability $P_{0,0,0}$ that the system is empty can be calculated from the normalizing condition P(0) + P(1) =1. $P_{0,0,0}$ is calculated from (30) and (31) .

Let L_s denote the average number of customers in the system, then we have

$L_s = L_{s_0} + L_{s_1}$
(32)

$$L_{s_0} = \left[\sum_{n=0}^{r} nP_{0,n,0} + \sum_{n=r+1}^{R-1} nP_{0,n,0}\right] + \left[\sum_{n=0}^{r}(n+1)P_{1,n,0} + \sum_{n=r+1}^{R-1}(n+1)P_{1,n,0}\right] \quad (33)$$

and

$$L_{s_1} = \left[\sum_{n=r+1}^{R} nP_{0,n,1} + \sum_{n=R+1}^{K} nP_{0,n,1}\right] + \left[\sum_{n=r+1}^{R}(n+1)P_{1,n,1} + \sum_{n=R+1}^{K}(n+1)P_{1,n,1}\right] \quad (34)$$

From (21) to (29) , (33) and (34) , we can calculate the value of L_s. The expected waiting time of the customers in the orbit is calculated as $W_s = \frac{L_s}{\bar{\lambda}}$, Where $\bar{\lambda} = \lambda_0 P(0) + \lambda_1 P(1)$. W_s is calculated from (30) to (32).

5. Numerical Illustrations

For various values λ_0, λ_1, μ, ϵ, θ, N while r, R, K, H_1, H_2 are fixed values, computed and tabulated the values of $P_{0,0,0}$,P(0), P(1), L_s and W_s.

TABLE 1

r	R	K	λ_0	λ_1	μ	θ	H_1	H_2	ϵ	N	$P_{0,0,0}$
4	7	10	4	2	4	2	0.9	0.1	0.5	10	2.5770156×10^{-4}
4	7	10	4	2	4	2	0.9	0.1	0.5	10	6.3247672×10^{-5}
4	7	10	5	3	4	2	0.9	0.1	0.5	10	5.0785361×10^{-5}
4	7	10	4	3	4	3	0.9	0.1	0.5	10	2.6044312×10^{-4}
4	7	10	4	3	4	3	0.9	0.1	0.5	11	9.5274241×10^{-5}
4	7	10	4	3	5	3	0.9	0.1	0.5	10	4.3289861×10^{-4}
4	7	10	3	2	4	2	0.9	0.1	1	10	1.6653298×10^{-4}
4	7	10	4	3	4	3	0.9	0.1	0.5	10	2.8568891×10^{-4}
4	7	10	3	2	4	2	0.9	0.1	0.5	10	2.8734389×10^{-4}

TABLE 2

P(0)	P(1)	L_s	W_s
0.289045964	0.712974235	5.076481742	2.209051472
0.049843111	0.950156894	5.668674173	2.757974617
0.048787671	0.971212328	6.229779643	2.01818703
0.132827476	0.987272524	3.209481271	0.99705507
0.217467401	0.794632599	5.262844644	1.655817655
0.440281789	0.579808311	2.789758843	0.77514678
0.049153509	0.940644382	6.047821001	2.965458747

0.325052513	0.694846577	3.096583413	0.96812372
0.273752025	0.745247975	6.204176795	2.739780419

Conclusion

It is observed from the tables 1 and 2 that when λ_0 increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase. When λ_1 increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase. When θ increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls andWs decrease. When θ increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls and Ws decrease. When μ increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls and Ws decrease. When N increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase.

References

[1] A.A. Bouchentouf, M. Kadi and A. Rabhi, Analysis of two heterogeneous server queueing model with balking, reneging and feedback, Mathematical sciences and application E-notes, 2(2) (2013) pp.10-21.
[2] B. Antline Nisha, and M. Thiagarajan, The M/M/1/K/N interdependent retrial queueing model with controllable arrival rates, International Journal of Mathematical Sciences and Engineering Application, ISSN 0973-9424, 8, N0.VI,(2014),pp. 27-41.
[3] B. Antline Nisha, and M. Thiagarajan, M/M/1/K interdependent retrial queueing model with impatient customers and controllable arrival rates, International journal of applied mathematical research (2014), Vol 3 (4), 485 – 495.
[4] G.I. Falin, A survey of retrial queues, Queuing Systems, 7(1990) pp. 127-168.
[5] G.I. Falin and J.G.C. Templeton, Retrial Queues, Chapman and Hall, London, (1997).
[6] J.R. Artalejo, A queueing sysytem with returning customers and waiting line, Operations Research Letters, 17(1995) pp. 191-199.
[7] J.R. Artalejo, Retrial queues with a finite number of sources, Journal of Korean Mathematical Society, 35(1998) pp. 503-525.
[8] J.R. Artalejo, A classified bibliography of research on retrial queues: Progress in 1990-1999 Top 7 (1999a) pp. 187-211.
[9] J.R. Artalejo, Accessible bibliography on retrial queues, Mathematical and Computer Modeling, 30(1999b) pp. 223-233.
[10] J.R. Artalejo, Accessible bibliography on retrial queues progress in 2000-2009, Mathematical Computer Modeling, 51(2010) pp. 1071-1081.
[11] M. Jain and A. Bhagat, Finite population retrial queueing model with threshold recovery geometric arrivals and impatient customers, Journal of Information and Operations Management, Vol 3(1), (2012) pp. 162-165.
[12] M. Thiagarajan and A. Srinivasan, The M/M/c/K/N interdependent queueing model with controllable arrival rates balking, reneging and spares, Journal of Statistics and Applications, 2, Nos.1-2,(2007) pp. 56-65.
[13] M.O. Abou-El-Ata and Hariri, A.M.A. The M/M/c/N queue with balking and reneging, Computers Ops. Res., 19, (1992) pp. 713-716.
[14] R. Kumar and S. Kumar Sharma, M/M/1/N Queuing System with Retention of Reneged Customers, Pakistan Journal of Statistics and Operation Research, 8(4), (2012), pp. 719-735.
[15] R. Kumar and S. Kumar Sharma, M/M/1/N Queuing System with Retention of Reneged Customers and balking, American Journal of Operational Research, 2(1), (2012), pp. 1-5.
[16] R. Kumar and S. Kumar Sharma, A single server markovian queueing system with discouraged arrivals and retention of reneged customers, Yugoslav Journal of Operation Research, 24(1) (2014) pp. 199-126.